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Radius-dependent gauge unification in AdS5

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Abstract: We examine the relation of the 4-dimensional low energy coupling of bulk gauge boson in a slice of AdS5 to the 5-dimensional fundamental couplings as a function of the orbifold radius R. This allows us to address the gauge coupling unification in AdS5 by means of the radius running as well as the conventional momentum running. Using the 4-dimensional effective supergravity approach, we compute the radius-dependent 1-loop couplings in generic AdS5 theory orbifolded by $Z_2 \times Z_2'$ with 4-dimensional supersymmetry, and discuss the low energy predictions when the 5-dimensional couplings are assumed to be unified.

Keywords: Renormalization Group, Field Theories in Higher Dimensions, Supersymmetric Effective Theories, GUT.

Contents

1. Introduction

It has been noted that the large scale hierarchy between the weak and Planck scales can be naturally obtained in 5-dimensional (5D) theory on a slice of AdS5 [1]. In the original model of Randall and Sundrum, all the standard model fields are assumed to be confined in the TeV brane. An apprarent drawback of this scenario is that one has to abandon the perturbative unification of gauge couplings at the fundamental scale of the model. An alternative scenario which may achieve gauge unification while solving the hierarchy problem is that the gauge fields propagate in 5D bulk spacetime [2]. In such case, the size of gauge coupling renormalization *calculable within the model* is generically of order $[\ln(M_{Pl}^2/M_W^2)]/8\pi^2$ [3]–[14], so the model can allow gauge unification at the 5D cutoff scale which is close to the 4-dimensional (4D) Planck scale M_{Pl}.

In this paper, we point out that the unification of bulk gauge couplings in 5D theories on a slice of AdS5 can be easily addressed if one could determine how the low energy gauge couplings vary under the change of the orbifold radius R. We then compute the R-dependent 1-loop 4D couplings in generic AdS5 theory orbifolded by $Z_2 \times Z_2'$ with $N = 1$ supersymmetry (SUSY) [15]. Our calculation scheme is the 4D effective supergravity (SUGRA) approach which has been discussed in [5] in which the R-dependent 1-loop couplings are computed for AdS5 models whose 5D vector multiplets are assumed to have $(++)$ boundary condition under $Z_2 : y \to -y$ and $Z_2' : y + \pi \to -y + \pi$. In this paper, we extend the analysis to the models containing 5D vector multiplets with generic $Z_2 \times Z_2'$ boundary conditions as well as the hypermultiplets with generic $Z_2 \times Z_2'$ boundary conditions.

2. Radius dependent gauge couplings in AdS5

Let us consider 5D gauge theory on a slice of AdS5 with orbifold radius R. The action includes

$$\int d^4x dy \sqrt{-G} \left[-\frac{1}{4g_{5a}^2} F^{aMN} F^a_{MN} - \left(\frac{\delta(y)}{\sqrt{G_{55}}} \frac{1}{g_{0a}^2} + \frac{\delta(y-\pi)}{\sqrt{G_{55}}} \frac{1}{g_{\pi a}^2} \right) \frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} \right], \qquad (2.1)$$

where $y = 0, \pi$ denote the 5-th coordinates of orbifold fixed points and the 5D metric G_{MN} is given by

$$ds^2 = G_{MN} dx^M dx^N = e^{-2kR|y|} g_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 \,, \tag{2.2}$$

where k is the AdS curvature and R is the orbifold radius. The massless 4D graviton $g_{\mu\nu}$ has R-dependent Planck scale:

$$M_{Pl}^2 = \frac{M_5^3}{k}(1 - e^{-2\pi kR}) \,,$$

where M_5 is the 5D Planck scale of G_{MN}. For generic bulk fields in AdS5, Kaluza-Klein (KK) scale is given by [2]

$$M_{KK} \simeq \frac{\pi k}{e^{\pi kR} - 1} \,. \tag{2.3}$$

In order for the background spacetime to be treated in semi-classical approximation, both k and $1/R$ are required not to exceed M_5. However still kR can take any value, either large or small, depending upon the radius stabilization mechanism. Obviously, in large radius limit $\pi kR \gg 1$, the KK scale is exponentially suppressed as $M_{KK} \approx \pi k e^{-\pi kR}$. On the other hand, in small radius limit with $\pi kR \lesssim 1$, the geometry is approximately flat and $M_{KK} \approx 1/R$.

Since 5D field theory is not renormalizable, the action (2.1) should be interpreted as an Wilsonian action with finite ultraviolet (UV) cutoff Λ. The quantity of our interest is the low energy coupling of zero mode gauge boson with external 4D momentum $p^2 \lesssim M_{KK}^2$ for generic value of kR. At tree approximation, they are simply given by

$$\left(\frac{1}{g_a^2}\right)_{\text{tree}} = \frac{\pi R}{g_{5a}^2} + \frac{1}{g_{0a}^2} + \frac{1}{g_{\pi a}^2} \,.$$

Including the 1-loop correction due to charged bulk and/or brane fields, the low energy couplings can be written as a function of p, R and k:

$$\frac{1}{g_a^2(p,R,k)} = \left[\frac{1}{g_{5a}^2(\Lambda)} + \frac{\gamma_a}{8\pi^3}\Lambda\right]\pi R + \left[\frac{1}{g_{0a}^2(\Lambda)} + \frac{1}{g_{\pi a}^2(\Lambda)} + \frac{b_a'}{8\pi^2}\ln\Lambda\right] + \frac{1}{8\pi^2}\tilde{\Delta}_a(p,R,k) \tag{2.4}$$

where $\Lambda \approx M_5$ is the cutoff scale of 5D theory measured by G_{MN} and $p^2 = -g^{\mu\nu}\partial_\mu\partial_\nu$ is measured by $g_{\mu\nu}$. Here the power-law divergence [16] is associated with the bulk counter term, while the log divergence is associated with the counter terms localized at orbifold fixed points [17, 18], and the conventional momentum-running is encoded in $\tilde{\Delta}_a$ together with finite KK threshold corrections. The coefficient γ_a of power-law divergence is highly sensitive to the used cutoff scheme, but the combination

$$\frac{1}{\hat{g}_{5a}^2} \equiv \frac{1}{g_{5a}^2(\Lambda)} + \frac{\gamma_a}{8\pi^3}\Lambda \tag{2.5}$$

is scheme-independent. Then the unification condition on 5D couplings, if exists any, should be imposed on the scheme-independent $\hat{g}_{5a}^2$.

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As for the couplings at fixed points, i.e. $g_{0a}^2(\Lambda)$ and $g_{\pi a}^2(\Lambda)$, they run logarithmically in Λ in such a way that $1/g_{0a}^2(\Lambda)+1/g_{\pi a}^2(\Lambda)+b'_a\ln\Lambda/8\pi^2$ is Λ-independent. Note that $g_{0a}^2(\Lambda)$ and $g_{\pi a}^2(\Lambda)$ correspond to the Wilsonian couplings defined at a common scale Λ in the metric frame of G_{MN}. However if we measure the scales in the metric frame of $g_{\mu\nu}$ as we do for p^2, then $g_{0a}^2(\Lambda)$ and $g_{\pi a}^2(\Lambda)$ should be interpreted as the Wilsonian couplings defined at different scales, Λ and $e^{-\pi kR}\Lambda$, respectively. At any rate, it is reasonable to assume [19] that

$$\frac{1}{g_{0a}^2(\Lambda)}\approx\frac{1}{g_{\pi a}^2(\Lambda)}=\mathcal{O}\left(\frac{1}{8\pi^2}\right). \tag{2.6}$$

Throughout this paper, we will adopt this strong coupling assumption and focus on the *parametrically large* 1-loop correction

$$\frac{1}{8\pi^2}\Delta_a^{(AdS)}=\frac{1}{g_a^2}-\left[\frac{\pi R}{\hat{g}_{5a}^2}+\frac{1}{g_{0a}^2}+\frac{1}{g_{\pi a}^2}\right]=\frac{1}{8\pi^2}\left[b'_a\ln\Lambda+\tilde{\Delta}_a\right], \tag{2.7}$$

while ignoring the subleading corrections of $\mathcal{O}(1/8\pi^2)$. Note that $\Delta_a^{(AdS)}$ is unambiguously calculable within 5D effective field theory.

Let us first summarize some generic features of the 1-loop correction Δ_a^{AdS}. The coefficients b'_a of log-divergence are scheme-independent as they are related with the coefficients of $\ln p$ and $\ln R$ which have infrared (IR) origin. Since the UV divergence structure is independent of the AdS curvature, b'_a can be computed in the flat limit $k=0$, yielding [17]

$$b'_a=\frac{1}{12}\left[\sum T_a(\phi^{++})-\sum T_a(\phi^{--})-23\sum T_a(A_M^{++})+23\sum T_a(A_M^{--})\right], \tag{2.8}$$

where $T_a(\Phi)=\mathrm{Tr}(T_a^2)$ for the gauge group representation Φ. Here $\phi^{zz'}$ and $A_M^{zz'}$ $(z,z'=\pm)$ denote 5D real scalar field and 5D vector field, respectively, with the $Z_2\times Z'_2$ orbifold boundary conditions: $\Phi^{zz'}(-y)=z\Phi^{zz'}(y)$, $\Phi^{zz'}(-y+\pi)=z'\Phi^{zz'}(y+\pi)$. When $p\lesssim M_{KK}$, the p-dependence of $1/g_a^2$ is given by

$$\frac{\partial\Delta_a^{(AdS)}}{\partial\ln p}=-b_a+\mathcal{O}(p^2/M_{KK}^2), \tag{2.9}$$

where b_a is the conventional 1-loop beta function coefficient in 4D effective theory, i.e.

$$b_a=\frac{1}{3}\sum T_a(\xi)+\frac{2}{3}\sum T_a(\psi)-\frac{11}{3}\sum T_a(A_\mu),$$

where ξ, ψ, and A_μ stand for 4D complex scalar, 4D Weyl fermion, and 4D real vector field, respectively. Also for $p\ll 1/R\ll\Lambda$ with $k=0$, one finds

$$(\Delta_a^{(AdS)})_{k=0}=b'_a\ln(\Lambda R)-b_a\ln(pR)+\mathcal{O}(1). \tag{2.10}$$

In the large radius limit $\pi kR\gg 1$, the p-dependence of $1/g_a^2$ can be determined within 5D field theory *only* for $p\lesssim e^{-k\pi R}\Lambda$. This can be easily seen for instance by considering the effects of higher-derivative terms in 5D action, e.g. $F^{aMN}G^{PQ}\partial_P\partial_Q F^a_{MN}/\Lambda$. Such term gives a contribution of order $p^2/e^{-2k\pi R}k\Lambda$ to $1/g_a^2$, which means that 5D field theory description *breaks down* for $p\gtrsim e^{-k\pi R}\sqrt{k\Lambda}$ [6]. So one can not probe a possible gauge

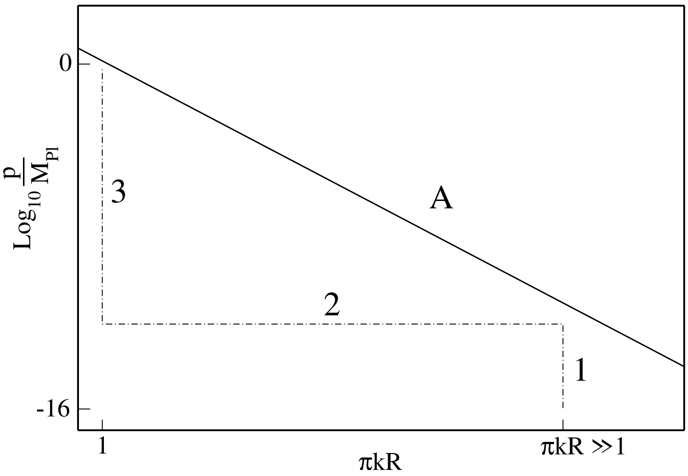

Figure 1: The domain of $[\ln p, \pi kR]$ which can be described by 5D effective field theory. The line A represents $\ln(p/M_{Pl}) \approx -\pi kR$.

unification at energy scales far above $e^{-k\pi R}\Lambda$ by means of the momentum-running alone. Physically, this is expected since the effective cutoff scale in the metric frame of $g_{\mu\nu}$ becomes y-dependent as $\Lambda_{\rm eff} \sim e^{-kR|y|}\Lambda$, while the gauge field zero mode is constant along the y-direction. As a result, the gauge field zero mode is required to have an external momentum $p \lesssim e^{-\pi kR}\Lambda$.

On the other hand, in the small radius limit $\pi kR \lesssim 1$, the leading p-dependence is calculable within the 5D field theory for the momentum range $p \lesssim \Lambda$. For $p \lesssim 1/R$ with $\pi kR \lesssim 1$, 4D effective theory calculation gives (2.9), while for $1/R \lesssim p \lesssim \Lambda$, 5D calculation yields

$$\frac{\partial \Delta_a^{(AdS)}}{\partial \ln p} \propto R\sqrt{p^2} + \mathcal{O}\left(\frac{1}{R^2p^2}, \frac{p^2}{\Lambda^2}, kR\right).$$

In particular, when $k \approx 1/\pi R \approx \Lambda$, we have

$$\left(\Delta_a^{(AdS)}\right)_{k\approx 1/\pi R\approx\Lambda} = b_a \ln\left(\frac{\Lambda}{p}\right) + \mathcal{O}(1)\,, \tag{2.11}$$

where the second term in the right side is generically of order one since there is no large scale ratio other than Λ/p in this case.

With the above observation, one efficient way to examine the gauge coupling unification in 5D theories on a slice of AdS is to see how the couplings vary under the change of the orbifold radius R as well as to examine the behavior under the change of external momentum p^2. To be explicit, let us consider a plane spanned by πkR and $\ln(p/M_{Pl})$ (figure 1). The range of $[\ln(p/M_{Pl}), \pi kR]$ which allows a 5D field theory description of physics is bounded to be below the line A representing $\ln(p/M_{Pl}) \approx -\pi kR$. Both the line 1 and the line 3 represent the conventional 4D momentum-running given by (2.9). For $\pi kR \gg 1$, the 4D momentum-running is allowed only for $p \lesssim e^{-\pi kR}\Lambda$ (the line 1), while for $\pi kR \lesssim 1$, it is allowed up to $p \lesssim \Lambda$ (the line 3) where $\Lambda = \mathcal{O}(M_{Pl})$ with $k \approx M_5 \approx M_{Pl}$. At $p \approx 1/\pi R \approx k \approx \Lambda$, we have $\Delta_a/8\pi^2 = 1/g_a^2 - \pi R/\hat{g}_{5a}^2 = \mathcal{O}(1/8\pi^2)$. So g_a^2 are unified at $p \approx 1/\pi R \approx k$ up to the corrections of order $1/8\pi^2$ if the 5D couplings $\hat{g}_{5a}^2$ are unified. From this unification point, one can move along the lines 3, 2, 1 to arrive at the phenomenologically relevant point with $p \approx M_W$ and $\pi kR \gg 1$. This procedure involves always a *radius-running* along the line 2 as well as the conventional 4D momentum-running along the lines 1 and 3. In the context of 5D theory, the gauge coupling unification would mean that the 5D coupling ratios $\hat{g}_{5a}^2/\hat{g}_{5b}^2$ are given by some *rational* numbers, e.g. the SU(5)-like unification $\hat{g}^2_{5_{\rm SU(3)}} = \hat{g}^2_{5_{\rm SU(2)}} = 5\hat{g}^2_{5_{\rm U(1)}}/3$. Such coupling unification can be a consequence of the unified 5D gauge group, or can appear as a matching condition at Λ even when the 5D gauge group is not a unified one. The resulting prediction on $g_a^2(p,R)$ at $p \approx M_W$ and $\pi kR \gg 1$ can be computed up to the subleading corrections of $\mathcal{O}(1/8\pi^2)$ if one could determine the R-dependence of $1/g_a^2$.

Even without radius running, there exist selective local quantities like Planck brane-brane propagator of gauge fields which are perfectively sensible up to k [6]. The radius running is complementary to the local observable discussions like Planck brane localized objects. Once the radius running is done, all the exclusive and inclusive quantities are valid up to very high energy scale and we do not need to worry about which observable is sensible.

3. 4D effective SUGRA calculation

It is possible to directly compute the R-dependent 1-loop gauge couplings in generic AdS5 theory [12]. However in cases with unbroken $N = 1$ SUSY, there is a much simpler way to compute the 1-loop couplings [5]. In such cases, the 4D effective action can be written as

$$S_{4D} = \int d^4x \left[\int d^4\theta \left\{ -3\exp\left(-\frac{K}{3}\right)\right\} + \left(\int d^2\theta \, \frac{1}{4} f_a \, W^{a\alpha} W^a_\alpha + P + \text{h.c.}\right)\right], \qquad (3.1)$$

where K is the real Kähler potential, f_a is the holomorphic gauge kinetic function for the a-th gauge superfield W^a_α, and P is the holomorphic superpotential. The Kähler potential can be expanded in powers of generic charged chiral superfield Φ:

$$K = K_0(T,T^*) + Z_\Phi(T,T^*)\Phi^* e^{-V}\Phi + \cdots ,$$

yielding

$$-3\exp\left(-\frac{K}{3}\right) = -3e^{-K_0(T,T^*)/3} + Y_\Phi(T,T^*)\Phi^* e^{-V}\Phi + \cdots , \qquad (3.2)$$

where T denotes generic gauge singlet moduli superfield, and $Y_\Phi = e^{-K_0/3} Z_\Phi$ for the Kähler metric Z_Φ. The 1-loop low energy gauge couplings are given by [20]

$$\begin{aligned} \frac{1}{g_a^2(p)} &= \text{Re}(f_a) + \frac{b_a}{16\pi^2}\ln\left(\frac{M_{Pl}^2}{e^{-K_0/3}p^2}\right) - \\ &\quad - \sum_\Phi \frac{T_a(\Phi)}{8\pi^2}\ln\left(Y_\Phi\right) + \frac{T_a(\text{Adj})}{8\pi^2}\ln\left(\text{Re}(f_a)\right) \\ &\equiv \text{Re}(f_a) + \frac{1}{8\pi^2}\Delta_a^{(4D)} , \end{aligned} \qquad (3.3)$$

where $b_a = \sum_\Phi T_a(\Phi) - 3T_a(\text{Adj})$ is the 1-loop beta function coefficient in 4D SUGRA, and M_{Pl} is the Planck scale of $g_{\mu\nu}$ which defines $p^2 = -g^{\mu\nu}\partial_\mu\partial_\nu$. Note that the moduli-dependence of M_{Pl}^2 differs in different metric frame which we use to measure p^2. For instance, in the 4D superconformal frame in which the action is given by (3.1), $M_{Pl}^2 \propto e^{-K_0/3}$, while in the 4D Einstein frame which is obtained from (3.1) after the Weyl scaling $g_{\mu\nu} \to e^{K_0/3} g_{\mu\nu}$, one has a field-independent M_{Pl}^2.

With (3.3), one can determine the 1-loop gauge couplings in AdS5 theory by computing f_a and K in the corresponding 4D effective SUGRA. In the procedure to match (3.3) onto the coupling (2.4) of underlying AdS5 theory, one should use $M_{Pl}^2 \propto e^{-K_0/3}$ since the 4D metric $g_{\mu\nu}$ in the AdS5 metric (2.2) corresponds to the metric in the 4D superconformal frame. Obviously, (3.3) indicates that one needs to know f_a *including the 1-loop correction*, while the *tree-level forms* of K_0 and Y_Φ are enough, to determine the 1-loop gauge couplings. Then it is convenient to decompose f_a as

$$f_a = \hat{f}_a + \frac{1}{8\pi^2}\Delta f_a \,, \tag{3.4}$$

where $\hat{f}_a$ is matched onto the uncalculable bare couplings of underlying AdS5 theory as

$$\text{Re}(\hat{f}_a) = \frac{\pi R}{\hat{g}_{5a}^2} + \frac{1}{g_{0a}^2(\Lambda)} + \frac{1}{g_{\pi a}^2(\Lambda)} \,,$$

where $\hat{g}_{5a}^2$, g_{0a}^2, and $g_{\pi a}^2$ are given in (2.5) and (2.6). With this, matching (3.3) onto (2.4) leads to

$$\text{Re}(\Delta f_a) = \Delta_a^{(AdS)} - \Delta_a^{(4D)} \,, \tag{3.5}$$

so Δf_a represents the finite 1-loop threshold correction to f_a which comes from integrating out massive KK modes.

In SUSY case, the radion R forms a $N = 1$ superfield T together with the axion B_5 being the 5-th component of the graviphoton B_M:

$$T = R + iB_5 \,,$$

and so

$$\hat{f}_a = \frac{\pi T}{\hat{g}_{5a}^2} + \mathcal{O}\left(\frac{1}{8\pi^2}\right).$$

Since it is finite and also p-independent at low momentum $p \ll M_{KK}$, $\text{Re}(\Delta f_a)$ of AdS5 theory is a function of k and R only, so Δf_a is a function of k and T only. In fact, one can find a field basis in which the relevant bulk interactions have a non-linear anomalous $\text{U}(1)_{PQ}$ symmetry under which $\delta_{PQ} B_5$ = real constant. This implies that $\delta_{PQ}(\Delta f_a)$ = imaginary constant. Together with the holomorphicity and a simple dimensional argument, $\text{U}(1)_{PQ}$ implies that

$$\frac{1}{8\pi^2}\Delta f_a = C_a \pi k T + \mathcal{O}\left(\frac{1}{8\pi^2}\right), \tag{3.6}$$

where C_a is a real constant. Then up to the subleading corrections of order one, Δf_a *can be determined simply by computing the loop-induced axion coupling* $B_5 F^{a\mu\nu}\tilde{F}^a_{\mu\nu}$. In the following, we will compute 1-loop gauge couplings in generic $N=1$ supersymmetric AdS5 theory using the 4D effective SUGRA approach which has been described in the above.

To proceed, let us consider a 5D SUGRA coupled super-Yang-Mills theory on $S^1/Z_2 \times Z_2'$ containing 5D vector and hypermultiplets as well as 3-brane fields localized at orbifold fixed points. The 5D action contains

$$
\begin{aligned}
S = \int d^5x\sqrt{-G}\Big[& \frac{M_5^3}{2}\left(\mathcal{R} - \frac{3}{4}C_{MN}C^{MN}\right) + \\
& + \frac{1}{\hat{g}_{5a}^2}\left(\frac{1}{2}D_M\phi^a D^M\phi^a - \frac{1}{4}F^{aMN}F^a_{MN} + \frac{i}{2}\bar{\lambda}^{ia}\gamma^M D_M\lambda^a_i\right) + \\
& + |D_M h^i_I|^2 + i\bar{\Psi}_I\gamma^M D_M\Psi_I + ic_I k\epsilon(y)\bar{\Psi}_I\Psi_I + \cdots\Big]
\end{aligned}
\tag{3.7}
$$

where $\mathcal{R}$ is the 5D Ricci scalar, $C_{MN} = \partial_M B_N - \partial_N B_M$ is the graviphoton field strength, ϕ^a, A^a_M and λ^{ia} $(i = 1, 2)$ are 5D scalar, vector and symplectic Majorana spinors constituting a 5D vector multiplet, h^i_I and Ψ_I are 5D scalar and Dirac spinor constituting the I-th hypermultiplet with kink mass $c_I k\epsilon(y)$. The AdS curvature k and kink mass c_I are related with the $\mathrm{U}(1)_R$-gauging with B_M in the following way:

$$
\begin{aligned}
D_M h^i_I &= \partial_M h^i_I - i\left(\frac{3}{2}(\sigma_3)^i_j - c_I\delta^i_j\right)k\epsilon(y)B_M h^j_I + \cdots \\
D_M\Psi_I &= \partial_M\Psi_I + ic_I k\epsilon(y)B_M\Psi_I + \cdots \\
D_M\lambda^{ai} &= \partial_M\lambda^{ai} - i\frac{3}{2}(\sigma_3)^i_j k\epsilon(y)B_M\lambda^{aj} + \cdots ,
\end{aligned}
\tag{3.8}
$$

where the ellipses stand for other gauge couplings. The 5D SUGRA multiplet is assumed to have the standard boundary condition under $Z_2 \times Z_2'$, leaving the 4D $N = 1$ SUSY unbroken. On the other hand, the vector and hypermultiplets can have arbitrary boundary conditions:

$$
\begin{aligned}
A^a_\mu(-y) &= z_a A^a_\mu(y)\,, & A^a_\mu(-y+\pi) &= z'_a A^a_\mu(y+\pi)\,, \\
h^i_I(-y) &= z_I(\sigma_3)^i_j h^j(y)\,, & h^i_I(-y+\pi) &= z'_I(\sigma_3)^i_j h^j_I(y+\pi)\,,
\end{aligned}
\tag{3.9}
$$

where $z_a, z'_a, z_I, z'_I = \pm 1$ and the boundary conditions of other 5D fields are fixed by (3.9) in the standard manner.

To derive the 4D effective SUGRA action, it is convenient to write the above 5D action in $N = 1$ superspace [21]. Here we will consider the most general case that there are brane gauge fields in addition to the bulk gauge fields of (3.7). So we introduce $N = 1$ gauge multiplets (A^h_μ, λ_h) and (A^l_μ, λ_l) living only on the UV (h = high) and IR (l = low) branes at $y = 0$ and π, respectively, and also chiral superfields $Q_h = (\phi_h, \psi_h)$ and $Q_l = (\phi_l, \psi_l)$ living only on the branes. Note that $Q_{h,l}$ can be charged for both bulk and brane gauge interactions. Among the fields in 5D gravity multiplet, we will keep only the radion superfield T and replace other gravity multiplets by their vacuum expectation values. Then following [21, 22], we find (in the unit with $M_5 = 1$)

$$
\begin{aligned}
S_1 = \int d^5x \Bigg[\int d^4\theta \Bigg\{ & \frac{1}{2}(T+T^*)e^{-(T+T^*)k|y|} \times \\
& \times \left(1 + e^{(\frac{3}{2}-c_I)(T+T^*)k|y|} H_I^* e^{-V} H_I + e^{(\frac{3}{2}+c_I)(T+T^*)k|y|} H_I^c e^V H_I^{c*}\right) + \\
& + \frac{2}{\hat{g}_{5a}^2} \frac{e^{-(T+T^*)k|y|}}{T+T^*} \left(\partial_y V^a - \frac{1}{\sqrt{2}}(\chi^a + \chi^{a*})\right)^2 \Bigg\} + \\
& + \int d^2\theta \left\{ \frac{1}{4\hat{g}_{5a}^2} T W^{a\alpha} W_\alpha^a + H_I^c \left(\partial_y - \frac{1}{\sqrt{2}}\chi\right) H_I + \text{h.c.} \right\} \Bigg], \\
S_2 = \int_{y=0} d^4x & \left[\int d^4\theta\, Q_h^* e^{-V} Q_h + \int d^2\theta \left(\frac{1}{4\hat{g}_h^2} W^{h\alpha} W_\alpha^h + P_h(Q_h, \tilde{H}_I) \right) + \text{h.c.} \right] + \\
+ \int_{y=\pi} d^4x & \Bigg[\int d^4\theta\, e^{-k\pi(T+T^*)} Q_l^* e^{-V} Q_l + \\
& + \int d^2\theta \left(\frac{1}{4\hat{g}_v^2} W^{l\alpha} W_\alpha^l + e^{-3k\pi T} P_l(Q_l, e^{(\frac{3}{2}-z_I' c_I)k\pi T} \tilde{H}_I') \right) + \text{h.c.} \Bigg],
\end{aligned}
\tag{3.10}
$$

where W_α^a is the chiral spinor superfield for the vector superfield V^a containing (A_μ^a, λ^a) with $\lambda^a = \frac{(1-\gamma_5)}{2}\lambda^{a1}$, and $W_\alpha^{h,l}$ stand for the 4D vector superfields $V^{h,l}$ containing the brane gauge multiplets $(A_\mu^{h,l}, \lambda_{h,l})$. In the metric frame of $g_{\mu\nu}$, the brane gauge couplings $\hat{g}_h^2$ and $\hat{g}_l^2$ correspond to the Wilsonian couplings at the scales Λ and $e^{-\pi kR}\Lambda$, respectively, for $\Lambda \approx M_5$. Here H_I and H_I^c are chiral superfields containing (h_I^1, ψ_I) and (h_I^{2*}, ψ_I^c), respectively, where $\psi_I = \frac{(1-\gamma_5)}{2}\Psi_I$, $\bar{\psi^c}_I = \frac{(1+\gamma_5)}{2}\Psi_I$. $\tilde{H}_I = H_I$ when $z_I = 1$, while $\tilde{H}_I = H_I^c$ when $z_I = -1$, and $\tilde{H}_I' = H_I$ when $z_I' = 1$, while $\tilde{H}_I' = H_I^c$ when $z_I' = -1$. Finally, χ^a are chiral superfields containing $(\phi^a + iA_5^a, \eta^a)$ where $\bar{\eta}^a = \frac{(1+\gamma_5)}{2}\lambda^{a1}$. The $Z_2 \times Z_2'$ boundary conditions of these $N = 1$ superfields are given by

$$
\begin{aligned}
V^a(-y) &= z_a V^a(y), & V^a(-y+\pi) &= z_a' V^a(y+\pi), \\
\chi^a(-y) &= -z_a \chi^a(y), & \chi^a(-y+\pi) &= -z_a' \chi^a(y+\pi), \\
H_I(-y) &= z_I H_I(y), & H_I(-y+\pi) &= z_I' H_I(y+\pi), \\
H_I^c(-y) &= -z_I H_I^c(y), & H_I^c(-y+\pi) &= -z_I' H_I^c(y+\pi).
\end{aligned}
\tag{3.11}
$$

Obviously, S_1 is from the 5D bulk terms in (3.7), while S_2 is the 4D brane action (with the superpotentials $P_{h,l}$) that was not written explicitly in (3.7). We have ignored the kinetic terms of bulk fields in the brane action S_2 by assuming that those terms are loop-suppressed compared to the bulk kinetic terms.

In fact, in order to write down the action in $N = 1$ superspace as (3.10), one needs to perform a R and B_5-dependent field redefintion. This results in a new piece of action arising from the chiral anomaly in orbifold field theory [23]. With its holomorphy property, the anomaly term can be simply determined by the following B_5-dependent transformation of fermions:

$$
\begin{aligned}
\lambda^{ai} &\to \left(e^{3ik|y|B_5\sigma_3/2}\right)^i_{\ j} \lambda^{aj}, & \Psi_I &\to e^{-ic_Ik|y|B_5}\Psi_I, \\
\psi_l &\to e^{-3ik\pi B_5/2}\psi_l, & \lambda_l &\to e^{3ik\pi B_5/2}\lambda_l,
\end{aligned}
\tag{3.12}
$$

which leads to

$$
\begin{aligned}
S_3 = \int d^5x \int d^2\theta \Big\{ & \frac{3}{4} T_a(\lambda^b)\left(z_b\delta(y) + z'_b\delta(y-\pi)\right) - \frac{1}{2} c_I T_a(\Psi_I)\left(z_I\delta(y) + z'_I\delta(y-\pi)\right) + \\
& + \frac{3}{2} T_a(\psi_l)\delta(y-\pi)\Big\} \frac{k|y|T}{16\pi^2} W^{a\alpha} W^a_\alpha + \\
+ \frac{3}{2}\left(T_l(\lambda_l) - T_l(\psi_l)\right) & \delta(y-\pi) \frac{k\pi T}{16\pi^2} W^{l\alpha} W^l_\alpha + \text{h.c.}\,,
\end{aligned} \tag{3.13}
$$

S_1 and S_2 have been derived recently in [22], however we are using different superfield basis for H_I and H_I^c, which is related to [22] by the holomorphic superfield redefinition: $H_I \to e^{-(\frac{3}{2}-c_I)Tk|y|} H_I$, $H_I^c \to e^{-(\frac{3}{2}+c_I)Tk|y|} H_I^c$. In our superfield basis, the y-independent modes of 5D superfields correspond to massless 4D superfields. In the following, let V^a to denote the 4D vector superfield being the zero mode of 5D vector multiplet $\mathcal{V}^a$ with $z_a = z'_a = 1$, H_I to be the 4D chiral superfield being the zero mode of 5D hypermultiplet $\mathcal{H}_I$ with $z_I = z'_I = 1$, H_I^c to be the 4D chiral superfield being the zero mode of 5D hypermultiplet $\mathcal{H}_I$ with $z_I = z'_I = -1$, and χ^a to be the 4D chiral superfield being the zero mode of 5D vector multiplet $\mathcal{V}^a$ with $z_a = z'_a = -1$. It is then straightforward to derive the following tree level Kähler metrics of H_I, H_I^c, Q_h, Q_l [22] and also the 4D Planck mass M_{Pl}:

$$
\begin{aligned}
Y_{H_I,H_I^c} &= \frac{M_5}{(\frac{1}{2} - z_I c_I)k}\left(e^{(\frac{1}{2}-z_I c_I)\pi k(T+T^*)} - 1\right), \\
Y_{Q_l} &= e^{-\pi k(T+T^*)}, \qquad Y_{Q_h} = 1\,, \\
M_{Pl}^2 &= e^{-K_0/3} M_5^2 = \frac{M_5^3}{k}\left(1 - e^{-k\pi(T+T^*)}\right),
\end{aligned} \tag{3.14}
$$

where $Y_\Phi = e^{-K_0/3} Z_\Phi$ for the Kähler metric Z_Φ of charged chiral superfield Φ. The calculation of the Kähler metric of χ^b (for $z_b = z'_b = -1$) is more involved as there is a contribution from the tadpole of A^b_μ which is given by

$$
A^b_\mu = \partial_\mu A^b_5 \left[y - \pi \frac{e^{(T+T^*)k|y|} - 1}{e^{(T+T^*)k\pi} - 1} \right].
$$

Including this tadpole, we find

$$
Y_{\chi^b} = \frac{k}{M_5} \frac{1}{e^{\pi k(T+T^*)} - 1}\,. \tag{3.15}
$$

As we have noticed, to obtain the 1-loop gauge couplings of V^A ($A = a, h, l$), one needs to know the holomorphic gauge kinetic functions f_A including the 1-loop threshold corrections (Δf_A) which can be determined by the loop-induced axion couplings $B_5 F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$. A nice feature of our superfield basis is that B_5 does *not* have any non-derivative interaction in S_1 other than the Chern-Simons coupling. As a result, integrating out massive KK modes does *not* generate 1-loop B_5-coupling to $F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$ other than those in S_3. This means that in our superfield basis, Δf_A can be simply determined by S_3 up to the subleading corrections of order one. We then find

$$
\begin{aligned}
f_a &= \frac{\pi T}{\hat{g}_{5a}^2} - \frac{3}{8\pi^2}\left(T_a(Q_l) - \frac{1}{2}\sum_b z_b' T_a(\mathcal{V}^a) + \frac{1}{3}\sum_I z_I' c_I T_a(\mathcal{H}_I)\right) k\pi T\,, \\
f_h &= \frac{1}{\hat{g}_h^2}\,, \qquad f_l = \frac{1}{\hat{g}_l^2} - \frac{3}{8\pi^2}\Big(T_l(Q_l) - T_l(V^l)\Big)\, k\pi T\,.
\end{aligned}
\tag{3.16}
$$

Applying (3.14) and (3.16) to (3.3), we easily find that the 1-loop brane couplings are given by

$$
\begin{aligned}
\frac{1}{g_h^2(p)} &= \frac{1}{\hat{g}_h^2} + \frac{1}{16\pi^2}(T_h(Q_h) - 3T_h(V^h))\ln\left(\frac{M_5^2}{p^2}\right), \\
\frac{1}{g_l^2(p,k,R)} &= \frac{1}{\hat{g}_l^2} + \frac{1}{16\pi^2}(T_l(Q_l) - 3T_l(V^l))\ln\left(\frac{e^{-2\pi kR}M_5^2}{p^2}\right),
\end{aligned}
\tag{3.17}
$$

as one would expect. The 1-loop bulk couplings at $p \lesssim M_{KK}$ can be obtained also by applying (3.14), (3.15) and (3.16) to (3.3):

$$
\begin{aligned}
\frac{1}{g_a^2(p,k,R)} = \frac{\pi R}{\hat{g}_{5a}^2} &+ \frac{1}{g_{0a}^2} + \frac{1}{g_{\pi a}^2} + \\
&+ \frac{1}{16\pi^2}\Bigg[T_a(Q_h)\ln\left(\frac{M_5^2}{p^2}\right) + T_a(Q_l)\ln\left(\frac{M_5^2 e^{-2\pi kR}}{p^2}\right) - \\
&\quad - T_a(\mathrm{Adj})\left(3\ln\left(\frac{M_5^2}{p^2}\right) - 3\pi kR - \ln\left(M_5^2R^2\right)\right) + \\
&\quad + \sum_{z_I = z_I'} T_a(\mathcal{H}_I)\left(\ln\left(\frac{k^2}{p^2}\right) - 2z_I c_I \pi kR - 2\ln\left(\frac{e^{(1-2z_Ic_I)\pi kR} - 1}{\pi(1-2z_Ic_I)}\right)\right) + \\
&\quad + \sum_{z_I = -z_I'} T_a(\mathcal{H}_I) 2 z_I c_I \pi kR - \sum_{z_b = -z_b'} T_a(\mathcal{V}^b) z_b 3\pi kR + \\
&\quad + \sum_{z_b = z_b' = -1} T_a(\mathcal{V}^b)\left(\ln\left(\frac{M_5^4}{p^2k^2}\right) + \pi kR + 2\ln(1 - e^{-2\pi kR})\right)\Bigg].
\end{aligned}
\tag{3.18}
$$

This result gives correct cutoff scales on the UV and IR branes, i.e. M_5 and $e^{-\pi kR}M_5$, in the metric frame of $g_{\mu\nu}$, and also the correct M_5-dependence with b_a' given by (2.8). In fact, the above result can be confirmed by a direct loop calculation in 5D theory which will be presented elsewhere [12].

4. Simple application and conclusion

As a simple example to show the effects of radius-running, let us consider a supersymmetric model in which the lepton superfields of the minimal supersymmetric standard model (MSSM) live on the UV brane, the MSSM quark superfields on the IR brane, the MSSM

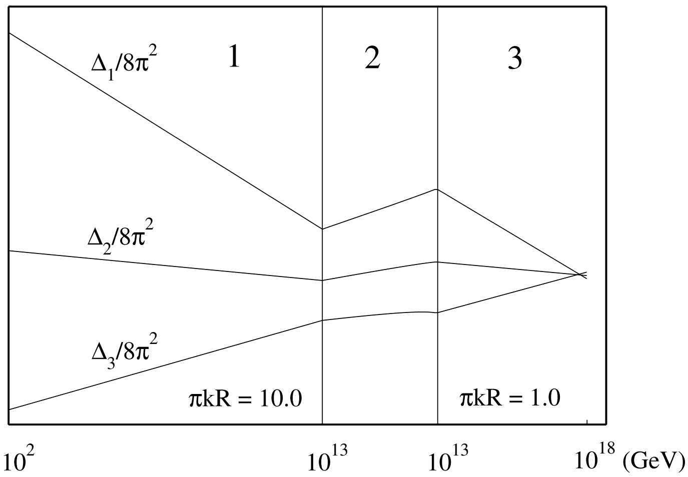

Figure 2: The running of 1-loop Δ_a^{AdS}. The region 1 represents the momentum running up to $p \approx 10^{13}$ GeV along the line 1 of figure 1, the region 2 is the radius running from $\pi kR = 10$ to $\pi kR = 1$ along the line 2, and the region 3 is the momentum running to $p \approx M_{Pl}$ along the line 3.

Higgs and gauge superfields in bulk. We choose $k = 5 \times 10^{17}$ GeV, $M_5 = 1.5 \times 10^{18}$ GeV, and assume the SU(5)-like unification of 5D couplings: $\hat{g}^2_{5_{\mathrm{SU(3)}}} = \hat{g}^2_{5_{\mathrm{SU(2)}}} = 5\hat{g}^2_{5_{\mathrm{U(1)}}}/3$. We also choose the kink masses of Higgs hypermultiplets given by $c_I = 1/2$, so that the Higgs zero modes are constant along y. As we have noted, the unification of $\hat{g}^2_{5a}$ implies that g_a^2 are unified at $p \approx 1/R \approx M_5$. If our universe has $1/R \approx M_5$, the 4D effective theory for $p \lesssim M_5$ is the MSSM, and then this model can not be compatible with the observed low energy couplings which indicate that gauge couplings are unified at 2×10^{16} GeV, not at $M_5 \approx 10^{18}$ GeV. However if our universe has $\pi kR \approx 10$, the momentum running along the line 1 is allowed only for $p \lesssim 7 \times 10^{13}$ GeV. One then has to include the effects of radius running along the line 2, making the observed low energy couplings to be consistent with the 5D unification at M_5 as depicted in figure 2. Our example aims to show the radius running can be sizable, but it does not provide any prediction about unification as the bulk group is not unified even at the cutoff scale.

To conclude, we have examined the orbifold radius R-dependent relation of low energy 4D couplings g_a^2 to the 5D fundamental couplings for bulk gauge bosons in a slice of AdS5. It is pointed out that the gauge coupling renormalization in AdS5 can be studied by means of the double running along $\ln p$ and R, as illustrated in figure 1. It is then crucial to compute the R-dependence of g_a^2 to address the issue of gauge unification in AdS5. Using the gauged $\mathrm{U}(1)_R$ and chiral anomaly in 5D SUGRA together with the holomorphy property of 4D effective SUGRA, we could compute the R-dependence of 1-loop 4D couplings in generic AdS5 theory with $N = 1$ SUSY. Our result (3.18) can be used to study gauge unification in supersymmetric AdS5 model.

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